Important Oblique Shock and Expansion Waves Formulas PDF

Formulas
Examples
with Units

List of 20
Important Oblique Shock and Expansion Waves Formulas

1) Expansion Waves Formulas

1.1) Flow Deflection Angle due to Expansion Wave Formula

Evaluate Formula

Formula

$$\theta_e = \left(\sqrt{\frac{\gamma_e + 1}{\gamma_e - 1}} \cdot atan\left(\sqrt{\frac{(\gamma_e - 1) \cdot (M_{e2}^2 - 1)}{\gamma_e + 1}} \right) - atan\left(\sqrt{M_{e2}^2 - 1} \right) \right) - \left(\sqrt{\frac{\gamma_e + 1}{\gamma_e - 1}} \cdot atan\left(\sqrt{\frac{(\gamma_e - 1) \cdot (M_{e1}^2 - 1)}{\gamma_e + 1}} \right) - atan\left(\sqrt{M_{e1}^2 - 1} \right) \right)$$

Example with Units

$$7.8669° = \left(\sqrt{\frac{1.41 + 1}{1.41 - 1}} \cdot atan\left(\sqrt{\frac{(1.41 - 1) \cdot (6^2 - 1)}{1.41 + 1}} \right) - atan\left(\sqrt{6^2 - 1} \right) \right) - \left(\sqrt{\frac{1.41 + 1}{1.41 - 1}} \cdot atan\left(\sqrt{\frac{(1.41 - 1) \cdot (5^2 - 1)}{1.41 + 1}} \right) - atan\left(\sqrt{5^2 - 1} \right) \right)$$

1.2) Flow Deflection Angle using Prandtl Meyer Function Formula

Evaluate Formula

Formula

$$\theta_e = \nu_{M2} - \nu_{M1}$$

Example with Units

$$6° = 83° - 77°$$

1.3) Forward Mach Angle of Expansion Fan Formula

Evaluate Formula

Formula

$$\mu_1 = arsin\left(\frac{1}{M_{e1}} \right)$$

Example with Units

$$11.537° = arsin\left(\frac{1}{5} \right)$$

1.4) Prandtl Meyer Function Formula

Evaluate Formula

Formula

$$\nu_M = \sqrt{\frac{\gamma_e + 1}{\gamma_e - 1}} \cdot atan\left(\sqrt{\frac{(\gamma_e - 1) \cdot (M^2 - 1)}{\gamma_e + 1}} \right) - atan\left(\sqrt{M^2 - 1} \right)$$

Example with Units

$$94.2021° = \sqrt{\frac{1.41 + 1}{1.41 - 1}} \cdot atan\left(\sqrt{\frac{(1.41 - 1) \cdot (8^2 - 1)}{1.41 + 1}} \right) - atan\left(\sqrt{8^2 - 1} \right)$$

1.5) Prandtl Meyer Function at Upstream Mach Number Formula

Evaluate Formula

Formula

$$v_{M1} = \sqrt{\frac{\gamma_e + 1}{\gamma_e - 1}} \cdot atan\left(\sqrt{\frac{(\gamma_e - 1) \cdot (M_{e1}^2 - 1)}{\gamma_e + 1}}\right) - atan\left(\sqrt{M_{e1}^2 - 1}\right)$$

Example with Units

$$75.9018° = \sqrt{\frac{1.41 + 1}{1.41 - 1}} \cdot atan\left(\sqrt{\frac{(1.41 - 1) \cdot (5^2 - 1)}{1.41 + 1}}\right) - atan\left(\sqrt{5^2 - 1}\right)$$

1.6) Pressure behind Expansion Fan Formula

Evaluate Formula

Formula

$$P_2 = P_1 \cdot \left(\frac{1 + 0.5 \cdot (\gamma_e - 1) \cdot M_{e1}^2}{1 + 0.5 \cdot (\gamma_e - 1) \cdot M_{e2}^2}\right)^{\frac{\gamma_e}{\gamma_e - 1}}$$

Example with Units

$$13.6106\,\text{Pa} = 40\,\text{Pa} \cdot \left(\frac{1 + 0.5 \cdot (1.41 - 1) \cdot 5^2}{1 + 0.5 \cdot (1.41 - 1) \cdot 6^2}\right)^{\frac{1.41}{1.41 - 1}}$$

1.7) Pressure Ratio across Expansion Fan Formula

Evaluate Formula

Formula

$$P_{e,r} = \left(\frac{1 + 0.5 \cdot (\gamma_e - 1) \cdot M_{e1}^2}{1 + 0.5 \cdot (\gamma_e - 1) \cdot M_{e2}^2}\right)^{\frac{\gamma_e}{\gamma_e - 1}}$$

Example

$$0.3403 = \left(\frac{1 + 0.5 \cdot (1.41 - 1) \cdot 5^2}{1 + 0.5 \cdot (1.41 - 1) \cdot 6^2}\right)^{\frac{1.41}{1.41 - 1}}$$

1.8) Rearward Mach Angle of Expansion Fan Formula

Evaluate Formula

Formula

$$\mu_2 = arsin\left(\frac{1}{M_{e2}}\right)$$

Example with Units

$$9.5941° = arsin\left(\frac{1}{6}\right)$$

1.9) Temperature behind Expansion Fan Formula

Evaluate Formula

Formula

$$T_2 = T_1 \cdot \left(\frac{1 + 0.5 \cdot (\gamma_e - 1) \cdot M_{e1}^2}{1 + 0.5 \cdot (\gamma_e - 1) \cdot M_{e2}^2}\right)$$

Example with Units

$$288.065\,\text{K} = 394.12\,\text{K} \cdot \left(\frac{1 + 0.5 \cdot (1.41 - 1) \cdot 5^2}{1 + 0.5 \cdot (1.41 - 1) \cdot 6^2}\right)$$

1.10) Temperature Ratio across Expansion Fan Formula

Evaluate Formula

Formula

$$T_{e,r} = \frac{1 + 0.5 \cdot (\gamma_e - 1) \cdot M_{e1}^2}{1 + 0.5 \cdot (\gamma_e - 1) \cdot M_{e2}^2}$$

Example

$$0.7309 = \frac{1 + 0.5 \cdot (1.41 - 1) \cdot 5^2}{1 + 0.5 \cdot (1.41 - 1) \cdot 6^2}$$

2) Oblique Shock Formulas

2.1) Component of Downstream Mach Normal to Oblique Shock Formula

Evaluate Formula

Formula

$$M_{n2} = M_2 \cdot \sin(\beta - \theta)$$

Example with Units

$$0.6661 = 1.21 \cdot \sin(53.4° - 20°)$$

2.2) Component of Downstream Mach Number Normal to Oblique Shock for given Normal Upstream Mach Number Formula

Evaluate Formula

Formula

$$M_{n2} = \sqrt{\frac{1 + 0.5 \cdot \left(\gamma_o - 1\right) \cdot M_{n1}^{2}}{\gamma_o \cdot M_{n1}^{2} - 0.5 \cdot \left(\gamma_o - 1\right)}}$$

Example

$$0.6666 = \sqrt{\frac{1 + 0.5 \cdot \left(1.4 - 1\right) \cdot 1.606^{2}}{1.4 \cdot 1.606^{2} - 0.5 \cdot \left(1.4 - 1\right)}}$$

2.3) Component of Upstream Mach Normal to Oblique Shock Formula

Evaluate Formula

Formula

$$M_{n1} = M_1 \cdot \sin\left(\beta\right)$$

Example with Units

$$1.6056 = 2 \cdot \sin\left(53.4^\circ\right)$$

2.4) Density behind Oblique Shock for given Upstream Density and Normal Upstream Mach Number Formula

Evaluate Formula

Formula

$$\rho_2 = \rho_1 \cdot \left(\left(\gamma_o + 1\right) \cdot \frac{M_{n1}^{2}}{2 + \left(\gamma_o - 1\right) \cdot M_{n1}^{2}}\right)$$

Example with Units

$$2.5012\,\text{kg/m}^3 = 1.225\,\text{kg/m}^3 \cdot \left(\left(1.4 + 1\right) \cdot \frac{1.606^{2}}{2 + \left(1.4 - 1\right) \cdot 1.606^{2}}\right)$$

2.5) Density Ratio across Oblique Shock Formula

Evaluate Formula

Formula

$$\rho_r = \left(\gamma_o + 1\right) \cdot \frac{M_{n1}^{2}}{2 + \left(\gamma_o - 1\right) \cdot M_{n1}^{2}}$$

Example

$$2.0418 = \left(1.4 + 1\right) \cdot \frac{1.606^{2}}{2 + \left(1.4 - 1\right) \cdot 1.606^{2}}$$

2.6) Flow Deflection Angle due to Oblique Shock Formula

Evaluate Formula

Formula

$$\theta = atan\left(\frac{2 \cdot \cot\left(\beta\right) \cdot \left(\left(M_1 \cdot \sin\left(\beta\right)\right)^{2} - 1\right)}{M_1^{2} \cdot \left(\gamma_o + \cos\left(2 \cdot \beta\right)\right) + 2}\right)$$

Example with Units

$$19.9888^\circ = atan\left(\frac{2 \cdot \cot\left(53.4^\circ\right) \cdot \left(\left(2 \cdot \sin\left(53.4^\circ\right)\right)^{2} - 1\right)}{2^{2} \cdot \left(1.4 + \cos\left(2 \cdot 53.4^\circ\right)\right) + 2}\right)$$

2.7) Pressure behind Oblique Shock for given Upstream Pressure and Normal Upstream Mach Number Formula

Evaluate Formula

Formula

$$P_b = P_a \cdot \left(1 + \left(\frac{2 \cdot \gamma_o}{\gamma_o + 1}\right) \cdot \left(M_{n1}^{2} - 1\right)\right)$$

Example with Units

$$166.2829\,\text{Pa} = 58.5\,\text{Pa} \cdot \left(1 + \left(\frac{2 \cdot 1.4}{1.4 + 1}\right) \cdot \left(1.606^{2} - 1\right)\right)$$

2.8) Pressure Ratio across Oblique Shock Formula

Evaluate Formula

Formula

$$P_r = 1 + \left(\frac{2 \cdot \gamma_o}{\gamma_o + 1}\right) \cdot \left(M_{n1}^{2} - 1\right)$$

Example

$$2.8424 = 1 + \left(\frac{2 \cdot 1.4}{1.4 + 1}\right) \cdot \left(1.606^{2} - 1\right)$$

2.9) Temperature behind Oblique Shock for given Upstream Temperature and Normal Upstream Mach Number Formula

Evaluate Formula

Formula

$$T_{s2} = T_{s1} \cdot \left(\frac{1 + \left(\frac{2 \cdot \gamma_o}{\gamma_o + 1}\right) \cdot \left(M_{n1}^{2} - 1\right)}{\left(\gamma_o + 1\right) \cdot \frac{M_{n1}^{2}}{2 + \left(\gamma_o - 1\right) \cdot M_{n1}^{2}}}\right)$$

Example with Units

$$400.9287\,\text{K} = 288\,\text{K} \cdot \left(\frac{1 + \left(\frac{2 \cdot 1.4}{1.4 + 1}\right) \cdot \left(1.606^{2} - 1\right)}{\left(1.4 + 1\right) \cdot \frac{1.606^{2}}{2 + \left(1.4 - 1\right) \cdot 1.606^{2}}}\right)$$

2.10) Temperature Ratio across Oblique Shock Formula

Evaluate Formula

Formula

$$T_r = \frac{1 + \left(\frac{2 \cdot \gamma_o}{\gamma_o + 1} \right) \cdot \left({M_{n1}}^2 - 1 \right)}{\left(\gamma_o + 1 \right) \cdot \frac{{M_{n1}}^2}{2 + \left(\gamma_o - 1 \right) \cdot {M_{n1}}^2}}$$

Example

$$1.3921 = \frac{1 + \left(\frac{2 \cdot 1.4}{1.4 + 1} \right) \cdot \left(1.606^2 - 1 \right)}{\left(1.4 + 1 \right) \cdot \frac{1.606^2}{2 + \left(1.4 - 1 \right) \cdot 1.606^2}}$$

Variables used in list of Oblique Shock and Expansion Waves Formulas above

- **M** Mach Number
- **M_1** Mach Number Ahead of Oblique Shock
- **M_2** Mach Number Behind Oblique Shock
- **M_{e1}** Mach Number Ahead of Expansion Fan
- **M_{e2}** Mach Number Behind Expansion Fan
- **M_{n1}** Upstream Mach Normal to Oblique Shock
- **M_{n2}** Downstream Mach Normal to Oblique Shock
- **P_1** Pressure Ahead of Expansion Fan *(Pascal)*
- **P_2** Pressure Behind Expansion Fan *(Pascal)*
- **P_a** Static Pressure Ahead of Oblique Shock *(Pascal)*
- **P_b** Static Pressure Behind Oblique Shock *(Pascal)*
- **$P_{e,r}$** Pressure Ratio Across Expansion Fan
- **P_r** Pressure Ratio Across Oblique Shock
- **T_1** Temperature Ahead of Expansion Fan *(Kelvin)*
- **T_2** Temperature Behind Expansion Fan *(Kelvin)*
- **$T_{e,r}$** Temperature Ratio Across Expansion Fan
- **T_r** Temperature Ratio Across Oblique Shock
- **T_{s1}** Temperature Ahead of Oblique Shock *(Kelvin)*
- **T_{s2}** Temperature Behind Oblique Shock *(Kelvin)*
- **v_{M1}** Prandtl Meyer Function at Upstream Mach no. *(Degree)*
- **v_{M2}** Prandtl Meyer Function at Downstream Mach no. *(Degree)*
- **β** Oblique Shock Angle *(Degree)*
- **γ_e** Specific Heat Ratio Expansion Wave
- **γ_o** Specific Heat Ratio Oblique Shock
- **θ** Flow Deflection Angle Oblique Shock *(Degree)*
- **θ_e** Flow Deflection Angle *(Degree)*
- **μ_1** Forward Mach Angle *(Degree)*
- **μ_2** Rearward Mach Angle *(Degree)*
- **v_M** Prandtl Meyer Function *(Degree)*
- **ρ_1** Density Ahead of Oblique Shock *(Kilogram per Cubic Meter)*
- **ρ_2** Density Behind Oblique Shock *(Kilogram per Cubic Meter)*
- **ρ_r** Density Ratio Across Oblique Shock

Constants, Functions, Measurements used in list of Oblique Shock and Expansion Waves Formulas above

- Functions: **arsin**, arsin(Number)
  Arcsine function, is a trigonometric function that takes a ratio of two sides of a right triangle and outputs the angle opposite the side with the given ratio.
- Functions: **atan**, atan(Number)
  Inverse tan is used to calculate the angle by applying the tangent ratio of the angle, which is the opposite side divided by the adjacent side of the right triangle.
- Functions: **cos**, cos(Angle)
  Cosine of an angle is the ratio of the side adjacent to the angle to the hypotenuse of the triangle.
- Functions: **cot**, cot(Angle)
  Cotangent is a trigonometric function that is defined as the ratio of the adjacent side to the opposite side in a right triangle.
- Functions: **sin**, sin(Angle)
  Sine is a trigonometric function that describes the ratio of the length of the opposite side of a right triangle to the length of the hypotenuse.
- Functions: **sqrt**, sqrt(Number)
  A square root function is a function that takes a non-negative number as an input and returns the square root of the given input number.
- Functions: **tan**, tan(Angle)
  The tangent of an angle is a trigonometric ratio of the length of the side opposite an angle to the length of the side adjacent to an angle in a right triangle.
- Measurement: **Temperature** in Kelvin (K)
  Temperature Unit Conversion
- Measurement: **Pressure** in Pascal (Pa)
  Pressure Unit Conversion
- Measurement: **Angle** in Degree (°)
  Angle Unit Conversion
- Measurement: **Density** in Kilogram per Cubic Meter (kg/m³)
  Density Unit Conversion

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